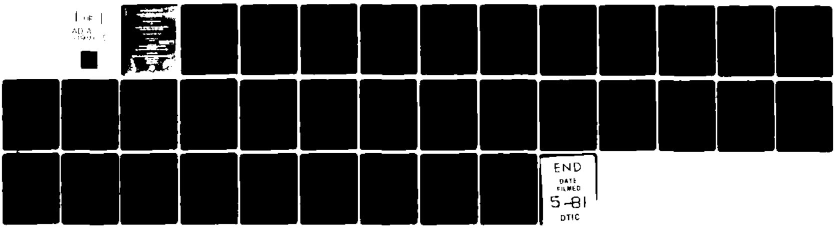
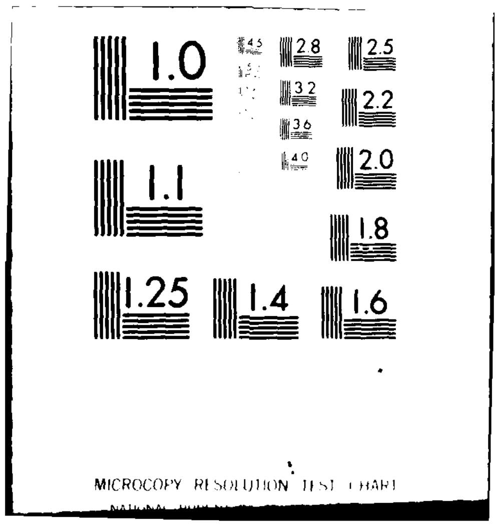
AD-A098 115 STANFORD UNIV CA INST FOR MATHEMATICAL STUDIES IN TH--ETC F/G 12/1
TWO PAPERS ON MAJORITY RULE: CONTINUITY PROPERTIES OF MAJORITY --ETC(U)
FEB 81 P COUGHLIN, S NITZAN, K LIN N00014-79-C-0685
UNCLASSIFIED TR-327 NL




1.0
4.5
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

TWO PAPERS ON MAJORITY RULE:

CONTINUITY PROPERTIES OF MAJORITY RULE
WITH INTERMEDIATE PREFERENCES.

~~by~~

~~Peter Coughlin and~~ Kuan-Pin Lin

~~and~~

ELECTORAL OUTCOMES WITH PROBABILISTIC VOTING
AND NASH SOCIAL WELFARE MAXIMA,

by

10 Peter Coughlin ~~and~~ Shmuel Nitzan Kuan Pin Lin

12 35

9 Technical ~~Report~~ No. 327

Feb~~ruary 1~~981

14 TR-327

A Report of the
Center for Research on Organizational Efficiency
Stanford University
15 Contract ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMIC SERIES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
Stanford, California
94305

114550

ABSTRACT

This technical report contains two joint (or co-authored) papers on aspects of majority rule.

The first (with Kuan-Pin Lin) studies the continuity properties of majority rule. Specifically, it shows that certain conditions which have previously been shown (by Grandmont) to be sufficient for a society's majority rule relation to be transitive or acyclic are also sufficient for the map from distributions of voter preferences to indices identified with their majority rule relations to be continuous. Applications of this result to societies with certain classical assumptions on preferences reveal that, in such societies, the map from distributions of voter preferences to their majority rule equilibria is also continuous.

The second (with Shmuel Nitzan) analyzes outcomes from electoral competitions with a Luce model of probabilistic voting. These outcomes are shown to be precisely the social alternatives that maximize a Nash-type social welfare function. These outcomes are also shown to be unanimity likelihood maxima when voting is independent. Finally, we show that the model's assumptions also imply the existence and uniqueness of electoral equilibria.

CONTINUITY PROPERTIES OF MAJORITY RULE WITH INTERMEDIATE PREFERENCES

by

Peter Coughlin
Institute for Mathematical Studies
in the Social Sciences
Fourth Floor, Encina Hall
Stanford University
Stanford, California 94305

Kuan-Pin Lin
School of System Sciences and
Department of Economics
Portland State University
P.O. Box 751
Portland, Oregon 97207

CONTINUITY PROPERTIES OF MAJORITY RULE
WITH INTERMEDIATE PREFERENCES*

by

Peter Coughlin and Kuan-Pin Lin

1. Introduction

Grandmont [1978] recently provided a general possibility theorem for majority rule. The theorem assumes two conditions on individual preferences (based on Chichilnisky [1976]) and one on the distribution of these preferences in a society (based on Davis, DeGroot and Hinich [1972]). It states that these three conditions imply that the society's majority rule relation is a particular ("median") alternative which is in the same family of individual relations as the domain of the society's distribution of individual preferences. This, in turn, implies that the society's majority rule relation is transitive or acyclic whenever this domain contains only transitive or acyclic individual preference relations. This paper shows that the assumptions in Grandmont [1978] <u>also</u> imply a continuity property for majority rule (Theorem 1).

The theorem in this paper is closely related to the results of Chichilnisky [1976], [1980 (a)-(b)]. She has shown that, with an <u>unrestricted</u> domain of preferences, it is <u>impossible</u> for a social choice rule which satisfies unanimity and anonymity (e.g. majority rule) to be continuous (viz., as a function from collections of individual preferences to a social preference). However, this paper proves, under the <u>same</u> domain restrictions which Grandmont

*This research was supported by the National Science Foundation and by Office of Naval Research Grant ONR-N0014-79-C-0685. We would also like to acknowledge helpful comments and suggestions from Ken Arrow, Graciela Chichilnisky, Richard Ericson, Steve Slutsky and Roy Radner. An earlier version of this paper was presented at the 1979 Econometric Society Meeting in Atlanta.

[1978] has shown to be sufficient for a transitive or acyclic majority rule relation, the method of majority decision is a continuous social choice rule (viz., as a function from distributions of indices identified with individuals preference relations to indices identified with their majority rule relations).

In Section 4, we apply our theorem to societies which satisfy certain classical assumptions on preferences which are included as special cases in Grandmont [1978], but which may have measure spaces of voters (as in Grandmont [1978]). In particular, we study societies with "quadratic-based preferences" and ones with the corresponding form of "single-peakedness." Theorem 1 implies that, in such societies, the map from distributions of voter preferences to their majority rule equilibria is also continuous (Corollaries 1 and 2). This provides continuity results which are analogous to the ones derived in Denzau and Parks [1975].

2. Grandmont's Model: Notation and Assumptions

Following Grandmont [1978]: X is a fixed set of alternatives on which each individual has a binary relation. $(R_a)_{a \in A}$ denotes a family of relations indexed by points, a, in an open convex subset A of E^n. This family is assumed to satisfy:

(H.1) (Weak Continuity): For every $x, y \in X$, the set $\{a \in A\colon\ xR_a y\}$ is closed in A.

(H.2) (Intermediate Preferences): For every $a', a'' \in A$, R_a is "between"[1/] $R_{a'}$ and $R_{a''}$ whenever $a = \lambda \cdot a' + (1-\lambda) \cdot a''$ for $\lambda \in (0,1)$.

A society is specified by a probability measure ν on A. Let A' and A" be the intersections of A with the two closed half-spaces determined by a hyperplane H. Every ν is assumed to satisfy:

(M.1) There exists $a^* \in A$ such that for every hyperplane H of E^n, $\nu(A') = \nu(A'')$ if and only if $a^* \in H$.

N(A) will denote the collection of probability measures on A which satisfy (M.1). The topology for N(A) will be the relative topology induced by the topology of weak convergence on the collection of all probability measures on A (e.g., see Billingsley [1968]).

The majority rule relation, R_M, for any society $\nu \in N(A)$ is given by xR_My if and only if $\nu\{a \in A: xR_ay\} \geq \nu\{a \in A: yR_ax\}$.

3. Continuity of Majority Rule

Grandmont [1978] showed that (H.1), (H.2) and (M.1) together imply that $R_M = R_{a*}$, where $a^* \in A$ satisfies the condition in (M.1). These three conditions therefore define the majority rule correspondence:

(1) $\phi(\nu) = \{a^* \in A | \forall\ H \subset E^n:\ \nu(A') = \nu(A'')$ if and only if $a^* \in H\}$

[1/] R_a is said to be "between" $R_{a'}$ and $R_{a''}$ if for all $x,y \in X$, (i) $xR_{a'}y$ and $xR_{a''}y$ imply xR_ay; (ii) $xP_{a'}y$ and $xP_{a''}y$ imply xP_ay; (iii) ($xI_{a'}y$ and $xP_{a''}y$) or ($xP_{a'}y$ and $xI_{a''}y$) imply xP_ay.

from each measure of voters, $\nu \in N(A)$, to a corresponding index or set of indices identified with the majority rule relation.

We prove the following in Section 5:

Theorem 1: Suppose that every society, $\nu \in N(A)$, satisfies (H.1), (H.2) and (M.1). Then the majority rule correspondence defined by (1) is a continuous function.

4. Applications to Majority Rule Equilibria

In this section we apply Theorem 1 to societies in which preferences satisfy classical assumptions from the papers preceding Grandmont [1978]. These applications provide results on the continuity properties of the function from distributions of voters' preferences to their majority rule voting equilibria (as in Denzau and Parks [1975]).

Suppose that R_M is the majority rule relation on the set of alternatives X for a particular society, ν. Then an alternative, $x \in X$, is a majority rule equilibrium (or Condorcet winner) for the society if and only if xR_My for every $y \in X$. That is, x cannot be defeated by a majority in a pairwise vote against any other alternative.

The assumptions in Grandmont [1978] assure that there is a majority rule equilibrium whenever the individual preferences are restricted to transitive or acyclic relations. However, even when there are majority rule equilibria, his assumptions do not assure that there is any nicely behaved relation between the indices and their maximal elements. However, in the following cases (which provided the standard representation of individual

preferences on public alternatives prior to Grandmont [1978]), it turns out that the map from distributions of voter preferences to their majority rule equilibria is also continuous.

4.a Quadratic-Based Preferences

Tullock [1969] assumed that there is a Euclidean policy space, $X \subset E^n$, and that the preference relation of each individual i satisfies

$$(2) \qquad xR_iy \text{ if and only if } \|x - x_i\| \leq \|y - x_i\|$$

for any $x,y \in X$ (where x_i is a unique "ideal point" and $\| \cdot \|$ is the usual Euclidean norm). (2) means that each individual ranks the possible policies according to their Euclidean distance from his ideal point. Such preferences have been labelled "Type I preferences" (e.g., Kramer [1977]). Such preferences can be completely specified by letting the index for each preference relation be its ideal point (i.e. $a = x_i \in E^n$).

The above preferences have been generalized to "ellipsoidal" or "quadratic-based" preferences (e.g., Davis, DeGroot, and Hinich [1972] or Riker and Ordeshook [1973]). Such preferences are given by a binary relation, R_i, which satisfies

$$(3) \qquad xR_iy \text{ if and only if } \|x - x_i\|_B < \|y - x_i\|_B$$

for any $x,y \in X$ (where x_i is a unique "ideal point" and $\|x\|_B = x' \cdot B \cdot x$ with B being a symmetric, positive definite, $(n \times n)$ matrix). This means that the indifference contours of an individual are ellipsoids. The ratio of the major axis to the minor axis of an

ellipsoidal indifference curve represents the relative salience of the dimensions. Given B, each preference relation can be completely specified by letting the index be the ideal point (i.e. $a = x_i \in E^n$).

Theorem 1 implies:

Corollary 1: Let $X \subset E^n$ and let B be given. Suppose that each citizen's preferences are indexed by his ideal point and satisfy (3). Then, if each society, ν, satisfies (M.1), the correspondence from each society to its majority rule equilibria is a continuous function.

4.b Single-Peaked Preferences

Black [1948], [1958] and Arrow [1963] have shown that "single-peakedness" implies the existence of a majority rule equilibrium. This condition requires a strong ordering of the alternatives such that, for each voter i, if xR_iy and y is between x and z (in the strong ordering) then yP_iz (e.g., see Arrow [1963], p. 77). This is most commonly formulated by assuming $X \subset E^1$ and

(4) xR_iy if and only if $\|x - x_i\| \leq \|y - x_i\|$

(as in (2)). In this case, voters' preferences are indexed by their ideal points.

We have

Corollary 2: Let $X \subset R^1$. Suppose that each citizen's preferences are indexed by an ideal point and satisfy (4). Then, if each society ν has a unique median, the correspondence from each society to its majority rule equilibria is a continuous function.

5. Proofs

In this section, we give proofs of the main theorem and its corollaries. We first develop properties of the majority rule correspondence ϕ defined by (1), culminating in the result that Grandmont's conditions (H.1), (H.2) and (M.1) imply that ϕ is a continuous function.

For notational convenience, denote a hyperplane which contains $a \in E^n$ by $H(a)$. The disjoint open half-spaces determinated by this hyperplane will be denoted by $H^+(a)$ and $H^-(a)$. Their closures will have the usual notation of $\overline{H}^+(a)$ and $\overline{H}^-(a)$.

Lemma 1: The majority rule map $\phi\colon N(A) \to A$ defined by (1) is a (single-valued) function.

Proof: From Grandmont's main theorem ([1978], p. 324), $\phi(\nu) = \{a^* \in A \mid \forall\ H(a) \subset E^n\colon\ \nu(A') = \nu(A'')$ if and only if $a^* \in H(a)\} \neq \emptyset$ for each $\nu \in N(A)$. Suppose that there exist $a, b \in A$, $a \neq b$, where a and b are both a^*'s for the same $\nu \in N(A)$. Since $a \neq b$, there is a family F of parallel hyperplanes such that $a \in H(a) \in F$ and $b \in H(b) \in F$ while $H(a) \neq H(b)$. Since a is an a^*, $\nu(A') = \nu(A'')$ for H only if $a \in H$. Since $H(a) \neq H(b)$, we have $a \notin H(b)$. Therefore, $\nu(A') \neq \nu(A'')$ for $H(b)$. But this contradicts b being an a^*. Hence $\phi(\nu)$ is single-valued for each $\nu \in N(A)$. Q.E.D.

Lemma 2: Let $H(b)$ and $H(c)$ be from the same family F of parallel hyperplanes in E^n with $H^+(b) \cap H^-(c) \neq \emptyset$. Then $a = \phi(\nu) \in H^+(b) \cap H^-(c)$ if and only if $\nu(H^+(b) \cap A) > 1/2$ and $\nu(H^-(c) \cap A) > 1/2$.

<u>Proof</u>: Suppose that $\nu(H^+(b) \cap A) > 1/2$ and $\nu(H^-(c) \cap A) > 1/2$, but $a = \phi(\nu) \notin H^+(b) \cap H^-(c)$. Then $\bar{H}^-(a) \subset E^n \backslash H^+(b)$ or $\bar{H}^+(a) \subset E^n \backslash H^-(c)$. Therefore, $\nu(\bar{H}^-(a) \cap A) \leq 1 - \nu(H^+(b) \cap A) < 1/2$ or $\nu(\bar{H}^+(a) \cap A) \leq 1 - \nu(H^-(c) \cap A) < 1/2$. But (M.1) implies that $\nu(\bar{H}^+(a) \cap A) = \nu(\bar{H}^-(a) \cap A) \geq 1/2$. A contradiction.

To show the converse, let $a \in H^+(b)$. Then $\bar{H}^+(a) \subset H^+(b)$. So $\nu(\bar{H}^+(a) \cap A) \leq \nu(H^+(b) \cap A)$. But (M.1) implies $\nu(\bar{H}^+(a) \cap A) \geq 1/2$. Therefore, $\nu(H^+(b) \cap A) \geq 1/2$. Suppose $\nu(H^+(b) \cap A) = 1/2$. Write $H^+(b) = [H^+(b) \cap H^-(a)] \cup \bar{H}^+(a)$. Then $1/2 = \nu(H^+(b) \cap A) = \nu(H^+(b) \cap H^-(a) \cap A) + \nu(\bar{H}^+(a) \cap A)$. But $\nu(\bar{H}^+(a) \cap A) \geq 1/2$ by (M.1). Therefore, $\nu(H^+(b) \cap H^-(a) \cap A) = 0$. Since A is open and convex, then $\nu(H^+(b) \cap H^-(a) \cap A) = 0$ says that there is some $d \in H^+(b) \cap H^-(a) \cap A$ with $\nu(A') = \nu(A'')$ for $H(d) \in F$. But then $\nu(A') = \nu(A'')$ does not occur only if $a \in H(a)$, which contradicts (M.1). Hence $\nu(H^+(b) \cap A) > 1/2$. A similar argument establishes $\nu(H^-(c) \cap A) > 1/2$ (see Figure 1 for the graphical interpretation of the proof for the case $a \in H^+(b)$ in E^2).

Q.E.D.

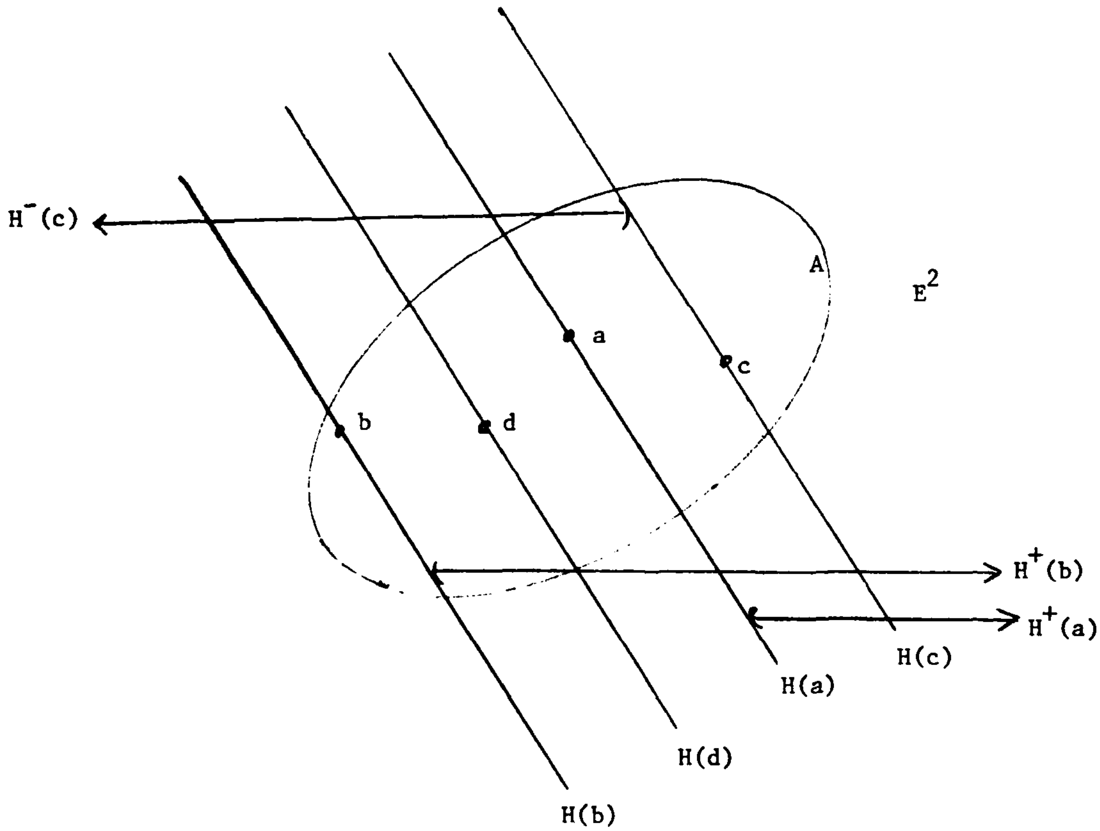


Figure 1

We are now in a position to prove our main theorem about the continuity of the majority rule correspondence defined by (1).

Proof of Theorem 1: First, by Lemma 1, ϕ is a single-valued function. Therefore, what we must prove is that for any a in the range of ϕ and any neighborhood of a, $U_\delta(a) = \{a' \in A: \|a - a'\| < \delta\}$ (with $\|\cdot\|$ being the Euclidean norm and δ a positive real number), the set $\phi^{-1}(U_\delta(a))$ is open in $N(A)$. For any such pair, choose any $\nu \in \phi^{-1}(U_\delta(a))$. A basic open neighborhood of ν is given by any set $B_\varepsilon(\nu) = \{\nu' \in N(A): \nu'(G_i) > \nu(G_i) - \varepsilon, i = 1,\ldots,k\}$ where the G_i are open and $\varepsilon > 0$. We will give $2n$ open sets G_i and show how to choose $\varepsilon > 0$ so that the resulting $B_\varepsilon(\nu)$ is contained in $\phi^{-1}(U_\delta(a))$.

First, the construction of the G_i's is as follows (see Figure 2 for the case $n = 2$): Since $U_\delta(a)$ is open, there exists an n-dimensional open set $I(b,c) = \{a' \in A: b_j < a'_j < c_j, j = 1,\ldots,n\}$ such that $\nu(I(b,c)) \neq 0$ and $a \in I(b,c) \subset U_\delta(a)$. Let β_j be the vector whose jth component is b_j while every other component is zero. Similarly, let γ_j be the vector whose jth component is c_j while every other component is zero. We now define the $G_i(i = 1,\ldots,k)$ as $G_{2j-1} = H^+(\beta_j) \cap A$ and $G_{2j} = H^-(\gamma_j) \cap A$, $j = 1,\ldots,n$, where $H(\beta_j)$ and $H(\gamma_j)$ are parallel to the hyperplane $\{y \in E^n: y_j = 0\}$ and $H^+(\beta_j) \cap H^-(\gamma_j) \neq \emptyset$. We notice that $k = 2n$.

Next, we will show that we can choose $\varepsilon > 0$ such that $B_\varepsilon(\nu) \subset \phi^{-1}(U_\delta(a))$. In particular, choose an $\varepsilon > 0$ which satisfies:

$$0 < \varepsilon < \min\left\{\nu(H^+(\beta_j) \cap A) - \frac{1}{2}\ ,\ \nu(H^-(\gamma_j) \cap A) - \frac{1}{2}\ ,\ j = 1,\ldots,n\right\} .$$

Such an ε exists by Lemma 2 since, by construction, $a \in H^+(\beta_j) \cap H^-(\gamma_j)$ for each $j = 1,\ldots,n$. Now, by definition, for any $\nu' \in B_\varepsilon(\nu)$, we have

$\nu'(H^+(\beta_j) \cap A) > \nu(H^+(\beta_j) \cap A) - \varepsilon$ and $\nu'(H^-(\gamma_j) \cap A) > \nu(H^-(\gamma_j) \cap A) - \varepsilon$ for $j = 1,\ldots,n$. From the chosen ε, then, $\nu'(H^+(\beta_j) \cap A) > 1/2$ and $\nu'(H^-(\gamma_j) \cap A) > 1/2$. By Lemma 2, the above inequalities imply that

$$d = \phi(\nu') \in H^+(\beta_j) \cap H^-(\gamma_j) \quad ,$$

i.e., $b_j < d_j < c_j$. Therefore, $d \in I(b,c) \subset U_\delta(a)$. Hence $\nu' \in \phi^{-1}(U_\delta(a))$. This shows that $B_\varepsilon(\nu) \subset \phi^{-1}(U_\delta(a))$. This means that every $\nu \in \phi^{-1}(U_\delta(a))$ is an interior point, so $\phi^{-1}(U_\delta(a))$ is open. Q.E.D.

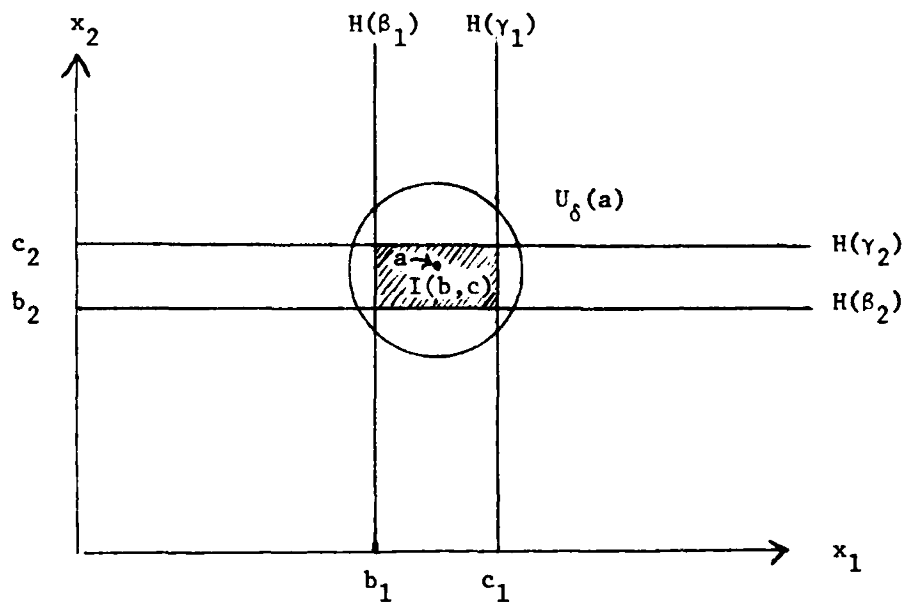


Figure 2

Proof of Corollary 1: Quadratic based preferences on a Euclidean policy space indexed by their ideal points satisfy (H.1) and (H.2). Therefore, ϕ: $N(A) \to A$ is continuous. Since, for $R_{a*} = R_M$, the index $a*$ is the unique majority rule equilibrium, the corollary follows. Q.E.D.

Proof of Corollary 2: Follows directly from Corollary 1.

References

Arrow, K., Social Choice and Individual Values, 2nd edition, New York: John Wiley and Sons [1963].

Billingsley, P., Convergence of Probability Measures, New York: John Wiley and Sons [1968].

Black, D., "On the Rationale of Group Decision Making," Journal of Political Economy, 56 [1948], 23-34.

Black, D., The Theory of Committees and Elections, Cambridge, England: Cambridge University Press [1958].

Chichilnisky, G., "Manifolds of Preferences and Equilibria," Ph.D. Dissertation, University of California, Berkeley, California [1976].

Chichilnisky, G., "Social Choice and the Topology of Spaces of Preferences," Advances in Mathematics, forthcoming [1980a].

Chichilnisky, G., "Structural Instability of Decisive Majority Rules," Journal of Mathematical Economics, forthcoming [1980b].

Davis, O., M. DeGroot and M. Hinich, "Social Preference Orderings and Majority Rule," Econometrica, 40 [1972], 147-157.

Denzau, A., and R. Parks, "The Continuity of Majority Rule Equilibrium," Econometrica, 43 [1975], 853-866.

Grandmont, J-M., "Intermediate Preferences and the Majority Rule," Econometrica, 46 [1978], 317-330.

Kramer, G., "A Dynamical Model of Political Equilibrium," Journal of Economic Theory, 16 [1977], 310-334.

Plott, C., "A Notion of Equilibrium and its Possibility Under Majority Rule," American Economic Review, 57 [1967], 787-806.

Riker, W., and P. Ordeshook, An Introduction to Positive Political Theory, Englewood Cliffs, New Jersey: Prentice-Hall [1973].

Tullock, G., "The General Irrelevance of the General Impossibility Theorem," Quarterly Journal of Economics, 81 [1967], 256-270.

ELECTORAL OUTCOMES WITH PROBABILISTIC VOTING AND NASH SOCIAL WELFARE MAXIMA*

by

Peter Coughlin
Institute for Mathematical Studies
in the Social Sciences
Fourth Floor, Encina Hall
Stanford University
Stanford, California 94305

Shmuel Nitzan
Department of Economics
Hebrew University of Jerusalem

*This research was supported by the National Science Foundation and the Office of Naval Research. We are also grateful for individual comments and suggestions from Ken Arrow, Mel Hinich, Bezalel Peleg, Micha Perles and Ariel Rubinstein. This paper was recently presented at the 1980 Public Choice Society Meeting in San Francisco.

ELECTORAL OUTCOMES WITH PROBABILISTIC VOTING AND NASH SOCIAL WELFARE MAXIMA

by

Peter Coughlin and Shmuel Nitzan

1. Introduction

Electoral competition and the maximization of a Nash social welfare function are two alternative methods of social decision making. The properties of Nash social welfare functions have recently been studied in Yaari [1978] and in Kaneko and Nakamura [1979]. Recent work on electoral competition has been concerned with the implications of probabilistic voting behavior. Some of the implications for full participation electorates have been developed in Comaner [1976], Hinich [1977, 1978] and Kramer [1978]. Related work on probabilistic abstention is due to Hinich, Ledyard and Ordeshook [1972, 1973] and Denzau and Kats [1977]. Additional social choice studies with probabilistic voting include Intriligator [1973, 1979], Nitzan [1975], Fishburn [1975] and Fishburn and Gehrlein [1977].

This paper studies spatial models of electoral competition with probabilistic voting where there is a very close connection between each voter's preferences and his choice probabilities. We show that in any such society

---

This research was supported by the National Science Foundation and by Office of Naval Research Grant ONR-N0014-79-C-0685. We would also like to acknowledge helpful comments and suggestions from Ken Arrow, Mel Hinich, Bezalel Peleg, Micha Perles and Ariel Rubenstein. An earlier version of this paper was presented at the 1980 Public Choice Society Meeting in San Francisco.

a policy is an electoral outcome if and only if it maximizes the society's Nash social welfare function (Theorem 1). An election is therefore a game which implements the Nash social welfare function in such societies. This result also implies that, when voting choices are made independently, electoral outcomes are policies which have the maximum likelihood of receiving unanimous support - and vice versa (Corollary 1). Additionally, Theorem 1 implies that there is always an equilibrium in pure strategies for the societies which we have studied (Corollary 2).

2. The Model

Societies with Probabilistic Voting

Let $X \subset R^m$ denote a non-empty compact and convex set of feasible social alternatives. The elements of X could be given different interpretations - such as alternative tax structures, alternative amounts of various public goods, alternative institutional arrangements or combinations of these. The individuals in a society are indexed by the $N = \{1,...,n\}$. Uncertainty in voters' behavior is represented by density functions, $f_i(x)$, on X for the individuals, $i \in N$. Each individual's probabilistic voting density function summarizes his choice probabilities. In particular, for any (measurable) subset $A \subset X$, $\int_A f_i(x) \cdot dx$ is the probability that individual i chooses some member of A (given that he can unilaterally determine the social choice). This generalizes the treatment of individual choice probatilities in Intriligator [1973, 1979] to Euclidean policy spaces.

In our discussion we follow the suggestion of Intriligator [1973, p. 553] that an individual's choice probabilities should be "proportional to his strength of preferences." To be precise, we assume that each individual's density function, f_i, is also his differentiable utility function, $U_i(x) > 0$ (up to a positive scalar multiple). This is referred to as a Luce model (e.g. Becker, DeGroot and Marschak [1963], Luce [1959] or Luce and Suppes [1965]). We also assume that these public sector preferences are concave (e.g. as in Hinich, Ledyard and Ordeshook [1972, 1973], Denzau and Kats [1977] and other studies).

Probabilistic Voting in Elections

In our elections there are two types of participants - individuals who vote and two candidates. Candidates try to win the support of voters by proposing alternatives which they will enact if elected. $\theta_1 \in X$ and $\theta_2 \in X$ denote the policies proposed by candidate 1 and candidate 2, respectively. $P_i^1(\theta_1,\theta_2)$ denotes the probability of individual i voting for candidate 1 when θ_1 and θ_2 have been proposed. $P_i^2(\theta_1,\theta_2)$ denotes the probability that i votes for candidate 2 under the same circumstances.

We consider electorates where everyone votes (as in Comaner [1976], Hinich [1977, 1978], Kramer [1978] and elsewhere). This means that

$$(1) \qquad P_i^1(\theta_1,\theta_2) + P_i^2(\theta_1,\theta_2) = 1 \quad .$$

We also assume that each individual's choice probabilities on every binary

set $\{\theta_1, \theta_2\}$ correspond to the likelihoods of his choosing these points from X. To be precise, we assume

(2) $$\frac{P_i^1(\theta_1,\theta_2)}{P_i^2(\theta_1,\theta_2)} = \frac{f_i(\theta_1)}{f_i(\theta_2)}$$

This merely says that the relative likelihoods of choosing θ_1 and θ_2 from X are preserved when the choice set consists only of θ_1 and θ_2. This is the continuous analog of the independence from irrelevant alternatives which follows from the basic choice axiom of Luce [1959] (see also Ray [1973]).

The individual choice probabilities on any pair of proposed policies are therefore given by

(3) $$P_i^1(\theta_1,\theta_2) = \frac{f_i(\theta_1)}{f_i(\theta_1) + f_i(\theta_2)} = \frac{U_i(\theta_1)}{U_i(\theta_1) + U_i(\theta_2)}$$

(4) and $$P_i^2(\theta_1,\theta_2) = \frac{f_i(\theta_2)}{f_i(\theta_1) + f_i(\theta_2)} = \frac{U_i(\theta_2)}{U_i(\theta_1) + U_i(\theta_2)}$$

This is a binary Luce model.

Electoral Outcomes

We are concerned with candidates (or political entrepreneurs) who are primarily office seekers. Such candidates will attempt to maximize their expected plurality (or margin of victory). We could, alternatively, assume that a candidate is seeking to maximize his probability of winning. However, Hinich [1977, pp. 212-213] has established that these two objectives are

equivalent when the electorate is large.

By (3) and (4), the expected plurality for candidate 1 is

$$P(\theta_1,\theta_2) = \sum_{i=1}^{n} \frac{f_i(\theta_1) - f_i(\theta_2)}{f_i(\theta_1) + f_i(\theta_2)} \quad . \tag{5}$$

By definition, the expected plurality for candidate 2 is $-P(\theta_1,\theta_2)$. We will let $M(\theta_1,\theta_2) = (P(\theta_1,\theta_2),-P(\theta_1,\theta_2))$ denote the candidates' payoff function.

When all of the above assumptions are satisfied we obtain the two-person, zero sum game,

$$\Gamma = \Gamma(X,X,M) \tag{6}$$

where X is the set of strategies available to each candidate, and M gives the candidates payoff functions. This game is referred to as the <u>electoral competition</u>.

An electoral equilibrium (or an equilibrium in pure strategies) for this game is a pair of proposed policies, $(\theta_1^*,\theta_2^*) \in X \times X$ which satisfies

$$P(\theta_1,\theta_2^*) \leq P(\theta_1^*,\theta_2^*) \leq P(\theta_1^*,\theta_2) \tag{7}$$

for every $\theta_1 \in X$ and $\theta_2 \in X$.

The actual outcome is determined by chance. However, since the game is symmetric and zero sum, we have the following two properties of Γ_1: first, $P(\theta_1^*,\theta_2^*) = 0$ at any electoral equilibrium (i.e., a tie is expected).

Second, if $\theta_1^* \neq \theta_2^*$ in an electoral equilibrium, then both (θ_1^*,θ_1^*) and (θ_2^*,θ_2^*) are pure strategy equilibria. We therefore refer to any policy in an equilibrium pair as an electoral outcome (or outcome).

Nash Social Welfare Maxima

Following Kaneko and Nakamura [1979] let each society have some distinguished alternative, x_o, which represents one of the worst alternatives for all individuals (the origin). Suppose x_o is excluded from the feasible set, X, being considered. The set $X^* = X \cup \{x_o\}$ is called the society's basic space of alternatives. The Nash social welfare function over the mixed alternatives (or lotteries) on X^* is

(8) $$W_o(U_i(x),\ldots,U_n(x)) = \sum_{i=1}^{n} \log\ (U_i(x) - U_i(x_o))$$

for individual utility functions, U_i, defined on these alternatives.

We are concerned with choices from the alternatives in X (rather than with lotteries on X^*). Additionally, we can set $U_i(x_o) = 0$ and have $U_i(x) > 0$ for every $x \in X$. The Nash social welfare function on X is then given by

(9) $$W(x) = \sum_{i=1}^{n} \log\ (U_i(x))$$

(as in Example 4.1 in Kaneko and Nakamura [1979]). A Nash social welfare maximum is any $x \in X$ which maximizes (9).

3. Results

Electoral Outcomes and Nash Social Welfare Maxima

We are now in a position to state our main theorem:

Theorem 1: An alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X,X,M)$ if, and only if, it is a Nash social welfare maximum over X.

The proof of this theorem is in the appendix.

The Probabilistic Unanimity Rule

Some possible alternatives to electoral competition and to the maximization of the Nash social welfare function have been based on unanimity rules. When voting is random, a natural extension of such rules can be given by the selection of an alternative which has the maximum likelihood of receiving unanimous support.

If individuals make independent voting decisions, i.e., their density functions, the f_i, are independent, then the likelihood that a policy x receives unanimous support (when voters can choose any policy in X) is

$$L(x) = \prod_{i=1}^{n} f_i(x) = \prod_{i=1}^{n} U_i(x) \tag{10}$$

for any $x \in X$.

We can now observe that the maxima of $L(x)$ are also maxima of

$$\log L(x) = \sum_{i=1}^{n} \log U_i(x) \tag{11}$$

which is the Nash social welfare function. Therefore we have:

Corollary 1: Suppose that individuals make independent voting decisions. Then an alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X,X,M)$ if, and only if, it is a unanimity likelihood maximum.

Existence of Electoral Equilibria

Theorem 1 has given a necessary and sufficient condition for an alternative to be an electoral outcome. Since the f_i (and hence the U_i) are continuous and the set of feasible alternative, X, is compact, this condition is always satisfied for some $x \in X$. Namely, there is always a Nash social welfare maximum. Hence,

Corollary 2: Every electoral competition $\Gamma(X,X,M)$ has an electoral equilibrium,

and clearly,

Corollary 3: Suppose that at least one individual has a strictly concave utility function. Then $\Gamma(X,X,M)$ has a unique electoral equilibrium.

These general existence results for global electoral equilibria do not use any special symmetry assumptions for the distribution of voter preferences (unlike Plott [1967], Sloss [1973], Matthews [1979] and others). They also follow without introducing abstentions and special assumptions about non-voting behavior (unlike Hinich, Ledyard and Ordeshook [1972, 1973] and Denzau and Kats [1977]).

4. Conclusion

This paper has studied societies with probabilistic voting. In particular, it has focused on societies where individual choice probabilities and preferences have a close relation so that a Luce model describes voting behavior. In these societies, policies are selected in an election if, and only if, they maximize the society's Nash social welfare function. If voting decisions are independent, such policies are also ones which are most likely to receive unanimous support when the society chooses from its entire set of feasible alternatives. Finally, these societies always have a global equilibrium, in candidate strategies (without including any of the special symmetry or non-voting assumptions of preceding papers).

APPENDIX

Proof of Theorem 1:

The theorem follows from lemmata 1-3.

Lemma 1: An alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X,X,M)$ if, and only if, it is a local maximum of $P(x,\theta)$ given the strategy $\theta_2 = \theta$

Proof: Γ_1 is a symmetric and zero-sum game. Therefore, if (θ_1,θ_2) with $\theta_1 \neq \theta_2$ is an electoral equilibrium, then (θ_1,θ_1) and (θ_2,θ_2) are electoral equilibria. Hence, $\theta \in X$ is an electoral outcome if, and only if, (θ,θ) is an electoral equilibrium. In other words, if, and only if,

$$P(x,\theta) \leq P(\theta,\theta) \leq P(\theta,y) \quad \forall\ x,y \in X \quad .$$

Put differently, $\theta \in X$ is an electoral outcome if, and only if, it is a global maximum of $P(x,\theta)$ for candidate 1 who chooses a strategy against $\theta_2 = \theta$.

Consider the payoff function,

$$P(x,\theta) = \sum_{i=1}^{n} \frac{f_i(x) - f_i(\theta)}{f_i(x) + f_i(\theta)} \quad .$$

Each term, $\frac{f_i(x) - f_i(\theta)}{f_i(x) + f_i(\theta)}$, is a strictly monotone increasing concave function of $f_i(x)$. Therefore, since $f_i(x) = U_i(x)$ is concave, each

term $\frac{f_i(x) - f_i(\theta)}{f_i(x) + f_i(\theta)}$ is a concave function of x. Consequently, since $P(x,\theta)$ is the sum of such terms, $P(x,\theta)$ is a concave function of x. Now, since every local maximum of a concave function on X is also a global maximum, the lemma follows. Q.E.D.

Lemma 2: An alternative, $\theta \in X$, is a global Nash social welfare maximum if, and only if, it is a local Nash social welfare maximum.

Proof: The Nash social welfare function is $W(x) = \sum_{i=1}^{n} \log U_i(x)$. Each term, $\log U_i(x)$, is a strictly monotone increasing concave function of $U_i(x)$. Therefore, since $U_i(x)$ is concave in x each term is concave in x. Consequently, since $W(x)$ is a sum of these terms, it is itself a concave function of x and the lemma is obtained. Q.E.D.

Lemma 3: An alternative, $\theta \in X$, is a local maximum of $P(x,\theta)$ if, and only if, it is a local maximum of $W(x)$.

Proof: Choose any $\theta \in X$. Then $b \in R^n$ is a permissible direction from θ if, and only if, there is some real number, $\lambda_1 > 0$, such that $(\theta + \lambda \cdot x) \in X$ for every $\lambda \in (0,\lambda_1)$. The alternative $\theta \in X$ is a local maximum of $P(x,\theta)$ if, and only if, the directional derivative, $D_bP(x,\theta)$ is non-positive for every permissible direction b. Additionally, $\theta \in X$ is a local maximum of $W(x)$ if, and only if, $D_bW(x) \leq 0$ for every permissible direction b.

Since each $f_i(x)$ is differentiable at every $x \in X$, we know that the directional derivatives of $P(x,\theta)$ and $W(x)$ are given by

$$D_b P(x,\theta)\Big]_{x=\theta} = \nabla P(x,\theta) \cdot b\Big]_{x=\theta}$$

and

$$D_b W(x)\Big]_{x=\theta} = \nabla W(x) \cdot b\Big]_{x=\theta}$$

for every permissible direction b. We therefore evaluate the partial derivatives of $P(x,\theta)$ and $W(x)$ to obtain

$$\frac{\partial P(x,\theta)}{x_h}\Bigg]_{x=\theta} = \sum_{i=1}^{n} \frac{2 \cdot f_i(\theta) \cdot \partial f_i(x)/\partial x_h}{[f_i(x) + f_i(\theta)]^2}\Bigg]_{x=\theta} = \sum_{i=1}^{n} \frac{\partial f_i(x)/\partial x_h\Big]_{x=\theta}}{2 \cdot f_i(\theta)}$$

and

$$\frac{\partial W(x)}{\partial x_h}\Bigg]_{x=\theta} = \sum_{i=1}^{n} \frac{\partial U_i(x)/\partial x_h}{U_i(x)}\Bigg]_{x=\theta} = \sum_{i=1}^{n} \frac{\partial f_i(x)/\partial x_h\Big]_{x=\theta}}{f_i(\theta)}$$

for $h = 1,\ldots,m$.

This now implies

$$D_b P(x,\theta)\Big]_{x=\theta} = \frac{1}{2} D_b W(x)\Big]_{x=\theta}$$

so that $D_b P(x,\theta) \leq 0$ if, and only if, $D_b W(x) \leq 0$ for every permissible direction b. Q.E.D.

References

Becker, G., M. DeGroot and J. Marschak, "Stochastic Models of Choice Behavior," Behavioral Science, 8 [1963], 41-55.

Comaner, W., "The Median Voter Rule and the Theory of Political Choice," Journal of Public Economics, 5 [1976], 169-177.

Denzau, A., and A. Kats, "Expected Plurality Voting Equilibrium and Social Choice Functions," Review of Economic Studies, 44 [1977], 227-233.

Fishburn, P., "A Probabilistic Model of Social Choice: Comment," Review of Economic Studies, 42 [1975], 297-301.

Fishburn, P., and W. Gehrlein, "Towards a Theory of Elections with Probabilistic Preferences," Econometrica, 45 [1977], 1907-1924.

Hinich, M., "The Mean Versus the Median in Spatial Voting Games" in Game Theory and Political Science, Ordeshook, ed., New York: New York University Press [1978].

Hinich, M., "Equilibrium in Spatial Voting: The Median Voter Result is an Artifact," Journal of Economic Theory, 16 [1977], 208-219.

Hinich, M., J. Ledyard and P. Ordeshook, "Non-voting and the Existence of Equilibrium under Majority Rule," Journal of Economic Theory, 4 [1972], 144-153.

Hinich, M., J. Ledyard and P. Ordeshook, "A Theory of Electoral Equilibrium: A Spatial Analysis Based on the Theory of Games," Journal of Politics, 35 [1973], 154-193.

Intriligator, M., "A Probabilistic Model of Social Choice," Review of Economic Studies, 40 [1973], 553-560.

Intriligator, M., "Income Redistribution: A Probabilistic Approach," American Economic Review, 69 [1979], 97-105.

Kaneko, M., and K. Nakamura, "The Nash Social Welfare Function," Econometrica, 47 [1979], 423-35.

Kramer, G., "Theories of Political Processes" in Frontiers of Quantitative Economics, III, New York: North-Holland [1977].

References Continued

Kramer, G., "Robustness of the Median Voter Result," Journal of Economic Theory, 19 [1978], 565-567.

Luce, R., Individual Choice Behavior, New York: Wiley [1959].

Luce, R., and P. Suppes, "Preference, Utility and Subjective Probability," in Handbook of Mathematical Psychology, Luce, Bush and Galanter, eds., New York: Wiley [1965].

Matthews, S., "A Simple Direction Model of Electoral Competition," Public Choice, 34 [1979], 141-156.

McKelvey, R., "Policy Related Voting and Electoral Equilibria," Econometrica, 43 [1975], 815-844.

Nitzan, S., "Social Preference Ordering in a Probabilistic Voting Model," Public Choice, 24 [1975], 93-100.

Owen, G., Game Theory, Philadelphia: Saunders [1968].

Plott, C., "A Notion of Equilibrium and its Possibility under Majority Rule," American Economic Review, 57 [1967], 787-806.

Ray, P., "Independence of Irrelevant Alternative," Econometrica, 41 [1973], 987-991.

Sloss, J., "Stable Outcomes in Majority Voting Games," Public Choice, 15 [1973], 19-48.

Yaari, M., "Rawls, Edgeworth, Shapley, Nash: Theories of Distributive Justice Re-Examined," Center for Research in Mathematical Economics and Game Theory, Research Memorandum No. 33 [1978].

REPORTS IN THIS SERIES

160. "The Structure and Stability of Competitive Dynamical Systems," by David Cass and Karl Shell.
161. "Monopolistic Competition and the Capital Market," by J. E. Stiglitz.
162. "The Corporation Tax," by J. E. Stiglitz.
163. "Measuring Returns to Scale in the Aggregate and the Scale Effect of Public Goods," by David A. Starrett.
164. "Monopoly, Quality, and Regulation," by Michael Spence.
165. "A Note on the Budget Constraint in a Model of Borrowing," by Duncan K. Foley and Martin F. Hellwig.
166. "Incentives, Risk, and Information: Notes Towards a *Theory of Hierarchy*," by Joseph E. Stiglitz.
167. "Asymptotic Expansions of the Distributions of Estimates in Simultaneous Equations for Alternative Parameter Sequences," by T. W. Anderson.
168. "Estimation of Linear Functional Relationships: Approximate Distributions and Connections with Simultaneous Equations in Econometrics," by T. W. Anderson.
169. "Monopoly and the Rate of Extraction of Exhaustible Resources," by Joseph E. Stiglitz.
170. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," by Michael Rothschild and Joseph Stiglitz.
171. "Strong Consistency of Least Squares Estimates in Normal Linear Regression," by T. W. Anderson and John B. Taylor.
172. "Incentive Schemes under Differential Information Structures: An Application to Trade Policy," by Partha Dasgupta and Joseph Stiglitz.
173. "The Incidence and Efficiency Effects of Taxes on Income from Capital," by John B. Shoven.
174. "Distribution of a Maximum Likelihood Estimate of a Slope Coefficient: The LIML Estimate for Known Covariance Matrix," by T. W. Anderson and Takamitsu Sawa.
175. "A Comment on the Test of Overidentifying Restrictions," by Joseph B. Kadane and T. W. Anderson.
176. "An Asymptotic Expansion of the Distribution of the Maximum Likelihood Estimate of the Slope Coefficient in a Linear Functional Relationship," by T. W. Anderson.
177. "Some Experimental Results on the Statistical Properties of Least Squares Estimates in Control Problems," by T. W. Anderson and John Taylor.
178. "A Note on "Fulfilled Expectations" Equilibria," by David M. Kreps.
179. "Uncertainty and the Rate of Extraction under Alternative Institutional Arrangements," by Partha Dasgupta and Joseph E. Stiglitz.
180. "Budget Displacement Effects of Inflationary Finance," by Jerry Green and E. Sheshinski.
181. "Towards a Marxist Theory of Money," by Duncan K. Foley.
182. "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities," by Sanford Grossman.
183. "*On the Efficiency of Competitive Stock Markets where Traders have Diverse Information*," by Sanford Grossman.
184. "A Bidding Model of Perfect Competition," by Robert Wilson.
185. "A Bayesian Approach to the Production of Information and Learning by Doing," by *Sanford J. Grossman*, Richard E. Kihlstrom and Leonard J. Mirman.
186. "Disequilibrium Allocations and Recontracting," by Jean-Michel *Grandmont, Guy Laroque and Yves Younes*.
187. "Agreeing to Disagree," by Robert J. Aumann.
188. "The Maximum Likelihood and the Nonlinear Three Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model," by Takeshi Amemiya.
189. "The Modified Second Round Estimator in the General Qualitative Response Model," by Takeshi Amemiya.
190. "Some Theorems in the Linear Probability Model," by Takeshi Amemiya.
191. "The Bilinear Complementarity Problem and Competitive Equilibria of Linear Economic Models," by Robert Wilson.
192. "*Noncooperative* Equilibrium Concepts for Oligopoly Theory," by L. A. Gerard-Varet.
193. "Inflation and Costs of Price Adjustment," by Eytan Sheshinski and Yoram Weiss.
194. "Power and Taxes in a Multi-Commodity Economy," by R. J. *Aumann and* M. Kurz.
195. "Distortion of Preferences, Income Distribution and the Case for a Linear Income Tax," by Mordecai Kurz.
196. "Search Strategies for Nonrenewable Resource Deposits," by Richard J. Gilbert.
197. "Demand for Fixed Factors, Inflation and Adjustment Costs," by Eytan Sheshinski and Yoram Weiss.
198. "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersions," by Steve Salop and Joseph Stiglitz.
199. "The Design of Tax Structure: Direct Versus Indirect Taxation by A. B. Atkinson and J. E. Stiglitz.
200. "Market Allocations of Location Choice in a Model with Free Mobility," by David Starrett.
201. "Efficiency in the Optimum Supply of Public Goods," by Lawrence J. Lau, Eytan Sheshinski and Joseph E. Stiglitz.
202. "Risk Sharing, Sharecropping and Uncertain Labor Markets," by David M. G. Newberry.
203. "On Non-Walrasian Equilibria," by Frank Hahn.
204. "A Note on Elasticity of Substitution Functions," by Lawrence J. Lau.
205. "Quantity Constraints as Substitutes for Inoperative Markets: The Case of the Credit Markets," by Mordecai Kurz.
206. "*Incremental Consumer's Surplus and Hedonic Price Adjustment*," by Robert D. Willig.

REPORTS IN THIS SERIES

207 "Optimal Depletion of an Uncertain Stock," by Richard Gilbert.

208 "Some Minimum Chi-Square Estimators and Comparisons of Normal and LogisticaModels in Qualitative Response Analysis," by Kimio Morimune

209 "A Characterization of the Optimality of Equilibrium in Incomplete Markets," by Sanford J. Grossman.

210 "Inflation and Taxes in a Growing Economy with Debt and Equity Finance," by M. Feldstein, J. Green and E. Sheshinski.

211 "The Specification and Estimation of a Multivariate Logit Model," by Takeshi Amemiya.

212 Prices and Queues as Screening Devices in Competitive Markets," by Joseph E. Stiglitz.

213 "Conditions for Strong Consistency of Least Squares Estimates in Linear Models," by T. W. Anderson and John B. Taylor.

214 "Utilitarianism and Horizontal Equity: The Case for Random Taxation," by Joseph E. Stiglitz.

215 "Simple Formulae for Optimal Income Taxation and the Measurement of Inequality," by Joseph E. Stiglitz.

216 "Temporal Resolution of Uncertainty and Dynamic Choice Behavior," by David M. Kreps and Evan L. Porteus.

217 "The Estimation of Nonlinear Labor Supply Functions with Taxes from a Truncated Sample," by Michael Hurd.

218 "The Welfare Implications of the Unemployment Rate," by Michael Hurd.

219. "Keynesian Economics and General Equilibrium Theory: Reflections on Some Current Debates," by Frank Hahn.

220. "The Core of an Exchange Economy with Differential Information," by Robert Wilson.

221. "A Competitive Model of Exchange," by Robert Wilson.

222. "Intermediate Preferences and the Majority Rule," by Jean-Michel Grandmont.

223. "The Fixed Price Equilibria: Some Results in Local Comparative Statics," by Guy Laroque.

224. "On Stockholder Unanimity in Making Production and Financial Decisions," by Sanford J. Grossman and Joseph E. Stiglitz.

225. "Selection of Regressors," by Takeshi Amemiya.

226. "A Note on A Random Coefficients Model," by Takeshi Amemiya.

227. "A Note on a Heteroscedastic Model," by Takeshi Amemiya.

228. "Welfare Measurement for Local Public Finance," by David Starrett.

229. "Unemployment Equilibrium with Rational Expectations," by W. P. Heller and R. M. Starr.

230. "A Theory of Competitive Equilibrium in Stock Market Economies," by Sanford J. Grossman and Oliver D. Hart.

231. "An Application of Stein's Methods to the Problem of Single Period Control of Regression Models," by Asad Zaman.

232. "Second Best Welfare Economics in the Mixed Economy," by David Starrett.

233. "The Logic of the Fix-Price Method," by Jean-Michel Grandmont.

234. "Tables of the Distribution of the Maximum Likelihood Estimate of the Slope Coefficient and Approximations," by T. W. Anderson and Takamitsu Sawa.

235. "Further Results on the Informational Efficiency of Competitive Stock Markets," by Sanford Grossman.

236. "The Estimation of a Simultaneous-Equation Tobit Model," by Takeshi Amemiya.

237. "The Estimation of a Simultaneous-Equation Generalized Probit Model," by Takeshi Amemiya.

238. "The Consistency of the Maximum Likelihood Estimator in a Disequilibrium Model," by T. Amemiya and G. Sen.

239. "Numerical Evaluation of the Exact and Approximate Distribution Functions of the Two-Stage Least Squares Estimate," by T. W. Anderson and Takamitsu Sawa.

240. "Risk Measurement of Public Projects," by Robert Wilson.

241. "On the Capitalization Hypothesis in Closed Communities," by David Starrett.

242. "A Note on the Uniqueness of the Representation of Commodity-Augmenting Technical Change," by Lawrence J. Lau.

243. "The Property Rights Doctrine and Demand Revelation under Incomplete Information," by Kenneth J. Arrow.

244. "Optimal Capital Gains Taxation Under Limited Information," by Jerry R. Green and Eytan Sheshinski.

245. "Straightforward Individual Incentive Compatibility in Large Economies," by Peter J. Hammond.

246. "On the Rate of Convergence of the Core," by Robert J. Aumann.

247. "Unsatisfactory Equilibria," by Frank Hahn.

248. "Existence Conditions for Aggregate Demand Functions: The Case of a Single Index," by Lawrence J. Lau.

249. "Existence Conditions for Aggregate Demand Functions: The Case of Multiple Indexes," by Lawrence J. Lau.

250. "A Note on Exact Index Numbers," by Lawrence J. Lau.

251. "Linear Regression Using Both Temporally Aggregated and Temporally Disaggregated Data," by Cheng Hsiao.

252. "The Existence of Economic Equilibria: Continuity and Mixed Strategies," by Partha Dasgupta and Eric Maskin.

253. "A Complete Class Theorem for the Control Problem and Further Results on Admissibility and Inadmissibility," by Asad Zaman.

254. "Measure-Based Values of Market Games," by Sergiu Hart.

255. "Altruism as an Outcome of Social Interaction," by Mordecai Kurz.

256. "A Representation Theorem for 'Preference for Flexibility'," by David M. Kreps.

257. "The Existence of Efficient and Incentive Compatible Equilibria with Public Goods," by Theodore Groves and John O. Ledyard.

258. "Efficient Collective Choice with Compensation," by Theodore Groves.

259. "On the Impossibility of Informationally Efficient Markets," by Sanford J. Grossman and Joseph E. Stiglitz.

REPORTS IN THIS SERIES

260. "Values for Games Without Sidepayments: Some Difficulties With Current Concepts," by Alvin E. Roth.
261. "Martingales and the Valuation of Redundant Assets," by J. Michael Harrison and David M. Kreps.
262. "Autoregressive Modelling and Money Income Causality Detection," by Cheng Hsiao.
263. "Measurement Error in a Dynamic Simultaneous Equations Model without Stationary Disturbances," by Cheng Hsiao.
264. "The Measurement of Deadweight Loss Revisited," by W. E. Diewert.
265. "The Elasticity of Derived Net Supply and a Generalized Le Chatelier Principle," by W. E. Diewert.
266. "Income Distribution and Distortion of Preferences: the ℓ Commodity Case," by Mordecai Kurz.
267. "The n^{-2} Order Mean Squared Errors of the Maximum Likelihood and the Minimum Logit Chi-Square Estimator," by Takeshi Amemiya.
268. "Temporal Von Neumann-Morgenstern and Induced Preferences," by David M. Kreps and Evan L. Porteus.
269. "Take-Over Bids and the Theory of the Corporation," by Stanford Grossman and Oliver D. Hart.
270. "The Numerical Values of Some Key Parameters in Econometric Models," by T. W. Anderson, Kimio Morimune and Takamitsu Sawa.
271. "Two Representations of Information Structures and their Comparisons," by Jerry Green and Nancy Stokey.
272. "Asymptotic Expansions of the Distributions of Estimators in a Linear Functional Relationship when the Sample Size is Large," by Naoto Kunitomo.
273. "Public Goods and Power," by R. J. Aumann, M. Kurz and A. Neyman.
274. "An Axiomatic Approach to the Efficiency of Non-Cooperative Equilibrium in Economies with a Continuum of Traders," by A. Mas-Colell.
275. "Tables of the Exact Distribution Function of the Limited Information Maximum Likelihood Estimator when the Covariance Matrix in Known," by T. W. Anderson and Takamitsu Sawa.
276. "Autoregressive Modeling of Canadian Money and Income Data," by Cheng Hsiao.
277. "We Can't Disagree Forever," by John D. Geanakoplos and Heraklis Polemarchakis.
278. "Constrained Excess Demand Functions," by Herklis M. Polemarchakis.
279. "On the Bayesian Selection of Nash Equilibrium," by Akira Tomioka.
280. "Disequilibrium Econometrics in Simultaneous Equations Systems," by C. Gourierous, J. J. Laffont and A. Monfort.
281. "Duality Approaches to Microeconomic Theory," by W. E. Diewert.
282. "A Time Series Analysis of the Impact of Canadian Wage and Price Controls," by Cheng Hsiao and Oluwatayo Fakiyesi.
283. "A Strategic Theory of Inflation," by Mordecai Kurz.
284. "A Characterization of Vector Measure Games in pNA," by Yair Tauman.
285. "On the Method of Taxation and the Provision of Local Public Goods," by David A. Starrett.
286. "An Optimization Problem Arising in Economics: Approximate Solutions, Linearity, and a Law of Large Numbers," by Sergiu Hart.
287. "Asymptotic Expansions of the Distributions of the Estimates of Coefficients in a Simultaneous Equation System," by Yasunori Fujikoshi, Kimio Morimune, Naoto Kunitomo and Masanobu Taniguchi.
288. "Optimal & Voluntary Income Distribution," by K. J. Arrow.
289. "Asymptotic Values of Mixed Games," by Abraham Neyman.
290. "Time Series Modelling and Causal Ordering of Canadian Money, Income and Interest Rate," by Cheng Hsiao.
291. "An Analysis of Power in Exchange Economies," by Martin J. Osborne.
292. "Estimation of the Reciprocal of a Normal Mean," by Asad Zaman.
293. "Improving the Maximum Likelihood Estimate in Linear Functional Relationships for Alternative Parameter Sequences," by Kimio Morimune and Naoto Kunitomo.
294. "Calculation of Bivariate Normal Integrals by the Use of Incomplete Negative-Order Moments," by Kei Takeuchi and Akimichi Takemura.
295. "On Partitioning of a Sample with Binary-Type Questions in Lieu of Collecting Observations," by Kenneth J. Arrow, Leon Pesotchinsky and Milton Sobel.
296.
297. "The Two Stage Least Absolute Deviations Estimators," by Takeshi Amemiya.
298. "Three Essays on Capital Markets," by David M. Kreps.
299. "Infinite Horizon Programs," by Michael J. P. Magill.
300. "Electoral Outcomes and Social Log-Likelihood Maxima," by Peter Coughlin and Shmuel Nitzan.
301. "Notes on Social Choice and Voting," by Peter Coughlin.
302. "Overlapping Expectations and Hart's Conditions for Equilibrium in a Securities Model," by Peter J. Hammond.
303. "Directional and Local Electorate Competitions with Probabilistic Voting," by Peter Coughlin and Shmuel Nitzan.
304. "Asymptotic Expansions of the Distributions of the Test Statistics for Overidentifying Restrictions in a System of Simultaneous Equations," by Kunitomo, Morimune, and Tsukuda.
305. "Incomplete Markets and the Observability of Risk Preference Properties," by H. H. Polemarchakis and L. Selden.
306. "Multiperiod Securities and the Efficient Allocation of Risk: A Comment on the Black-Scholes Option Pricing Model," by David M. Kreps.
307. "Asymptotic Expansions of the Distributions of k-Class Estimators when the Disturbances are Small," by Naoto Kunitomo, Kimio Morimune, and Yoshihiko Tsukuda.
308. "Arbitrage and Equilibrium in Economies with Infinitely Many Commodities," by David M. Kreps.
309. "Unemployment Equilibrium in an Economy with Linked Prices," by Mordecai Kurz.
310. "Pareto Optimal Nash Equilibria are Competitive in a Repeated Economy," by Mordecai Kurz and Sergiu Hart.
311. "Identification," by Cheng Hsiao.
312. "An Introduction to Two-Person Zero Sum Repeated Games with Incomplete Information," by Sylvain Sorin.

Reports in this Series

313. "Estimation of Dynamic Models With Error Components," by T. W. Anderson and Cheng Hsiao.

314. "On Robust Estimation in Certainty Equivalence Control," by Anders H. Westlund and Hans Stenlund.

315. "On Industry Equilibrium Under Uncertainty," by J. Drèze and E. Sheshinski.

316. "Cost Benefit Analysis and Project Evaluation From the Viewpoint of Productive Efficiency" by W. E. Diewert.

317. "On the Chain-Store Paradox and Predation: Reputation for Toughness," by David M. Kreps and Robert Wilson.

318. "On the Number of Commodities Required to Represent a Market Games," Sergiu Hart.

319. "Evaluation of the Distribution Function of the Limited Information Maximum Likelihood Estimator," by T. W. Anderson, Naoto Kunitomo, and Takamitsu Sawa.

320. "A Comparison of the Logit Model and Normal Discriminant Analysis When the Independent Variables Are Binary," by Takeshi Amemiya and James L. Powell.

321. "Efficiency of Resource Allocation by Uninformed Demand," by Theodore Groves and Sergiu Hart.

322. "A Comparison of the Box-Cox Maximum Likelihood Estimator and the Nonlinear Two Stage Least Squares Estimator," by Takeshi Amemiya and James L. Powell.

323. "Comparison of the Densities of the TSLS and LIMLK Estimators for Simultaneous Equations," by T. W. Anderson, Naoto Kunitomo, and Takamitsu Sawa.

324. "Admissibility of the Bayes Procedure Corresponding to the Uniform Prior Distribution for the Control Problem in Four Dimensions but Not in Five," by Charles Stein and Asad Zaman.

325. "Some Recent Developments on the Distributions of Single-Equation Estimators," by T. W. Anderson.

326. "On Inflation", by Frank Hahn

327. Two Papers on Majority Rule: "Continuity Properties of Majority Rule with Intermediate Preferences," by Peter Coughlin and Kuan-Pin Lin; and, "Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Maxima," by Peter Coughlin and Shmuel Nitzan.

ND

DATE

LMED